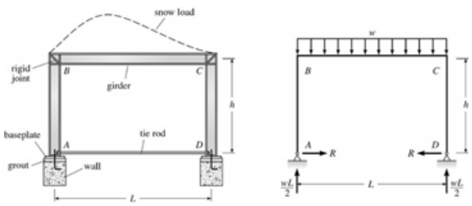
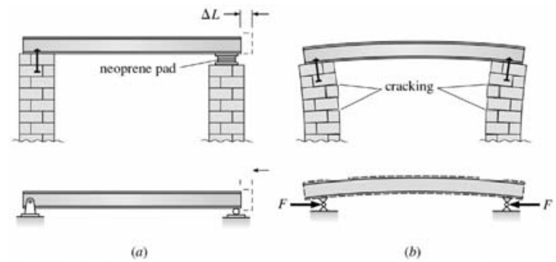
Analysis of Statically Determinate Structures

- The most common type of structure an engineer will analyze lies in a plane subject to a force system in the same plane.



Analysis of Statically Determinate Structures



Analysis of Statically Determinate Structures

- In general, it is not possible to perform an exact analyze of a structure.
- Approximations for structure geometry, material parameters, and loading type and magnitude must be made.
- **Support connections** - Structural members may be joined in a variety of methods, the most common are ***pin*** and ***fixed*** joints

Analysis of Statically Determinate Structures

- A **pin** connection confines deflection; allows rotation
- A **fixed** connection confines deflection and rotation
- However, in reality, a pin connection has some resistance against rotation due to friction, therefore, a **torsional spring** connection may be more appropriate. If the stiffness $k = 0$ the joint is a **pin**, if $k = \infty$, the joint is **fixed**.

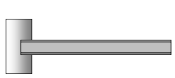
Support Idealizations

- A **pin** connection confines deflection; allows rotation

Pin support

Roller support

- A **fixed** connection confines deflection and rotation

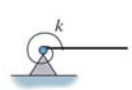


Fixed support

Support Idealizations

- However, in reality, a pin connection has some resistance against rotation due to friction, therefore, a **torsional spring** connection may be more appropriate. If the stiffness $k = 0$ the joint is a **pin**, if $k = \infty$, the joint is **fixed**.

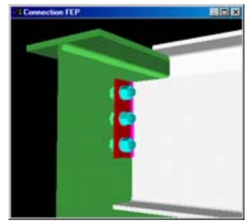
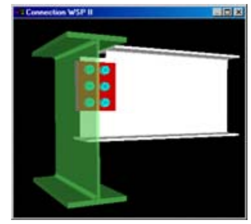
Torsional spring support



Support Idealizations

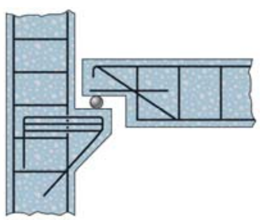
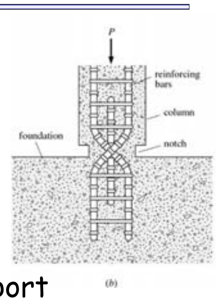

Pin support

Support Idealizations

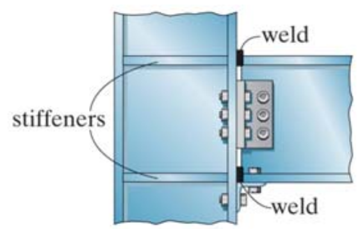
Pin support

Support Idealizations

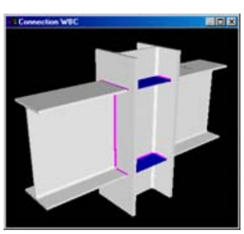
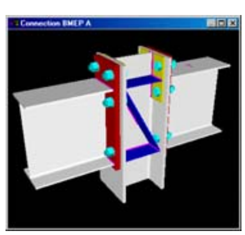
Pin support

Support Idealizations



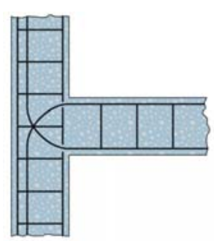
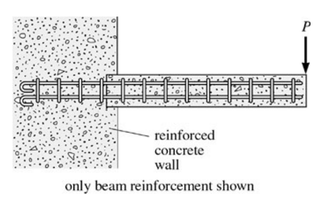
Fixed support

Support Idealizations

Fixed support

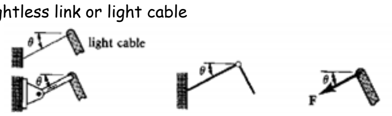
Support Idealizations

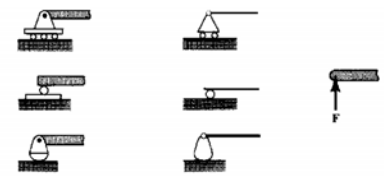
Fixed support

Support Idealizations

- Weightless link or light cable



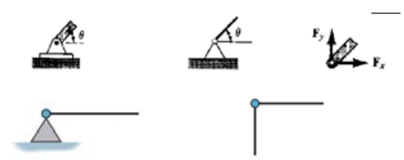
- Rollers and rockers



Support Idealizations

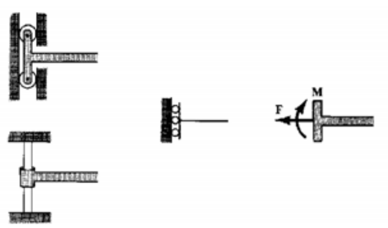
- Smooth contacting surface

- Smooth pin or hinge



Support Idealizations

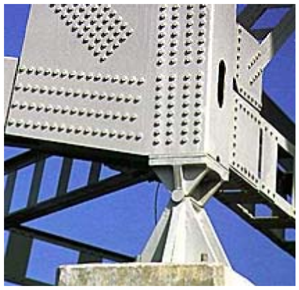
- Sliders and collar



- Fixed support

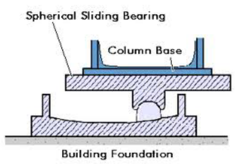
Support Idealizations

- Smooth pin



Support Idealizations

- New friction pendulum bearings on the I-40 bridge



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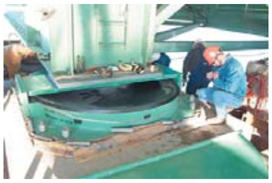
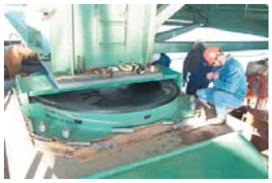


Support Idealizations

- New friction pendulum bearings on the I-40 bridge



Support Idealizations

- New friction pendulum bearings on the I-40 bridge

Support Idealizations

- Smooth pin

Support Idealizations

- Smooth hinge

Support Idealizations

- Smooth hinge

Support Idealizations

- Smooth hinge

Support Idealizations

- Roller support

Support Idealizations

- Roller support

Support Idealizations

- Fixed support

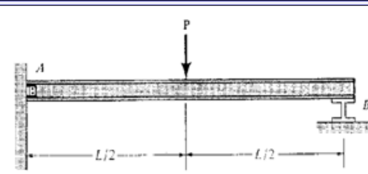
Support Idealizations

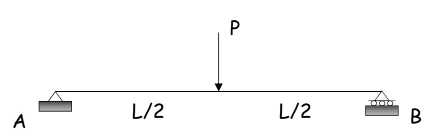
- Fixed support

Support Idealizations

- A complex structure may be idealized as a *line drawing* where orientation of members and type of connections are assumed.
  - In many cases, loadings are transmitted to a structure under analysis by a secondary structure.
  - In a *line drawing,* a pin support is represented by lines that do not touch and a fixed support by connecting lines

Support Idealizations





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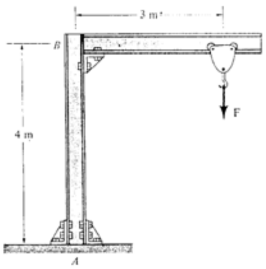

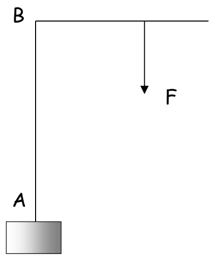

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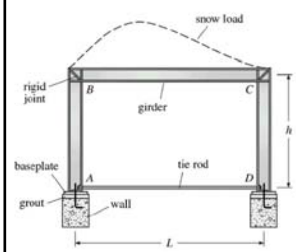

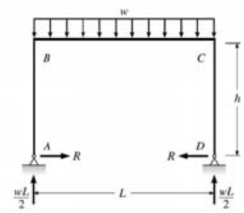

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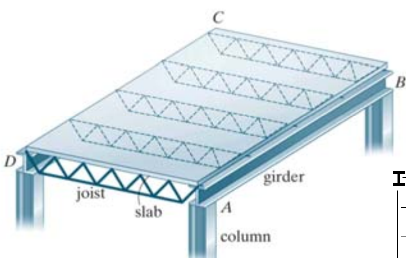

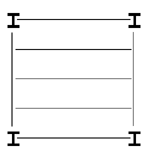

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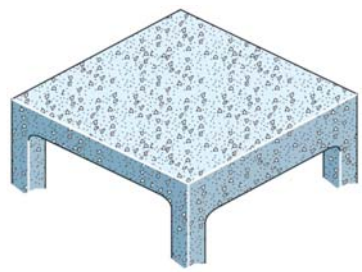

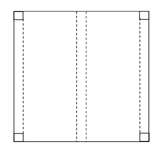

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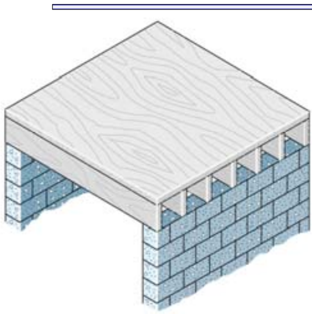

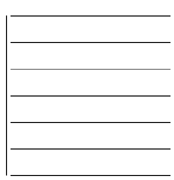

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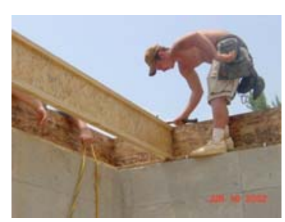
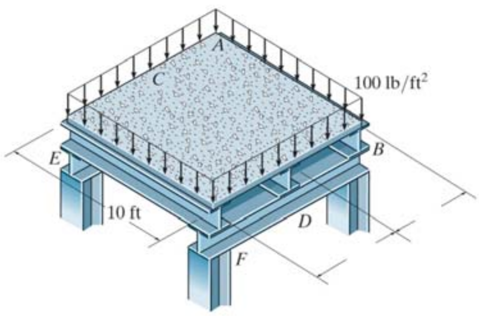


Support Idealizations

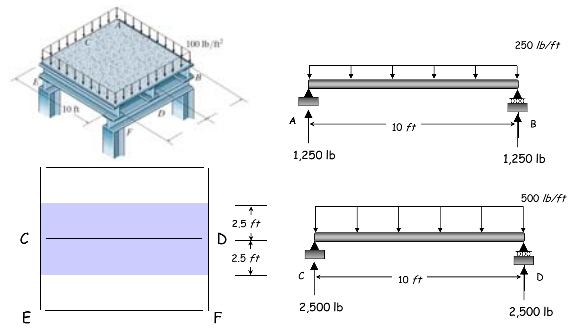
Loading Idealizations

- **Tributary Loadings** - When frames or other structural members are analyzes, it is necessary to determine how walls, floors, or roofs transmit load to the element under consideration.
  - A **one-way system** is typically a slab or plate structure supported along two opposite edges
  - Examples, a slab of reinforced concrete with steel in one direction or a with steel in both directions with a span ratio $L_2/L_1 > 2$
  - A **two-way system** is typically defined by a span ratio $L_2/L_1 < 2$ or if the all edges are supported

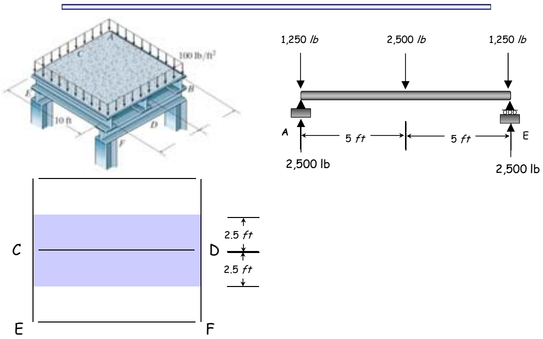
Loading Idealizations



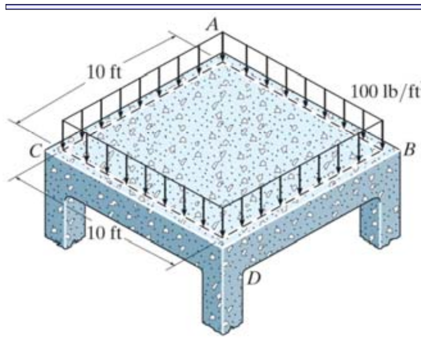
Loading Idealizations



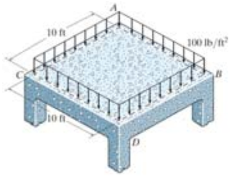
Loading Idealizations

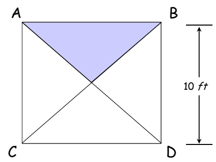


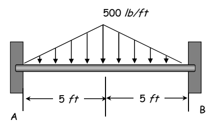
Loading Idealizations



Loading Idealizations







Principle of Superposition

- Basis for the theory of linear elastic structural analysis:

  The total displacement or stress at a point in a structure subjected to several loadings can be determined by adding together the displacements or stresses caused by each load acting separately.

- There are two exceptions to these rule:

  If the material does not behave in a linear-elastic manner

  If the geometry of the structure changes significantly under loading (example, a column subjected to a bucking load)

Equations of Equilibrium

- From statics the equations of equilibrium are:

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- However, since are dealing with co-planar structures the equations reduce to

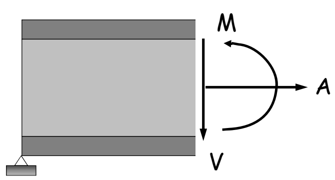
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0$$

Equations of Equilibrium

- In order to apply these equations, we first must draw a **free-body diagram (FBD)** of the structure or its members.
- If the body is isolated from its supports, all forces and moments acting on the body are included.

Equations of Equilibrium

- If **internal loadings** are desired, the method of sections is used.
- A FBD of the cut section is used to isolate internal loadings.
- In general, internal loadings consist of an axial force **A**, a shear force **V**, and the bending moment **M**.



Determinacy and Stability

- **Determinacy** - provide both necessary and sufficient conditions for equilibrium.
  - When all the forces in structure an be determined from the equations of equilibrium then the structure is considered ***statically determinate***.
  - If there are more unknowns than equations, the structure is ***statically indeterminate***.

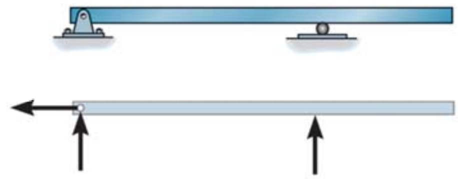
Determinacy and Stability

- For co-planar structures, there are three equations of equilibrium for each FBD, so that for ***n*** bodies and ***r*** reactions:

r = 3n statically determinate

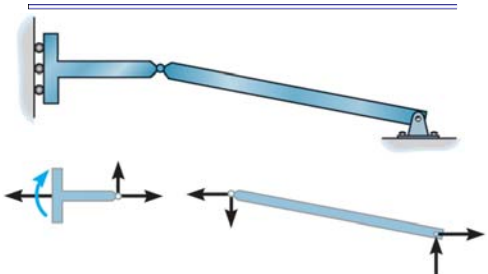
r > 3n statically indeterminate

Determinacy and Stability



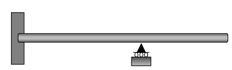
r = 3
r=3n determinate
n = 1

Determinacy and Stability

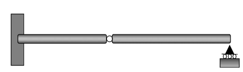


r = 6
r=3n determinate
n = 2

Determinacy and Stability

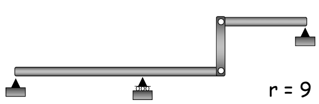


r = 4
r>3n indeterminate
n = 1



r = 6
r=3n determinate
n = 2

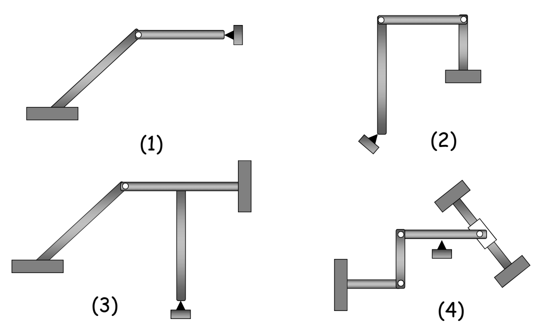
Determinacy and Stability



r = 9
r=3n determinate
n = 3

r = 10
r>3n indeterminate
n = 3

Determinacy and Stability



Determinacy and Stability

- **Stability** - Structures must be properly held or constrained by their supports
  - **Partial Constraints** - a structure or one of its member with fewer reactive forces then equations of equilibrium
  - **Improper Constraints** - the number of reactions equals the number of equations of equilibrium, however, all the reactions are concurrent. In this case, the moment equations is satisfied and only two valid equations of equilibrium remain.

Determinacy and Stability

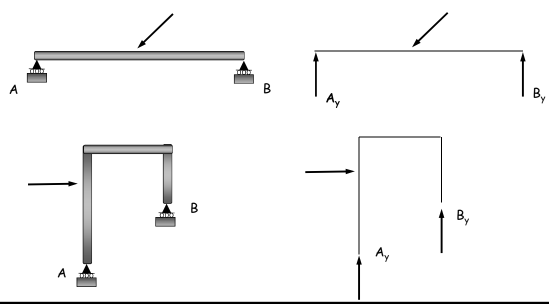
- Another case is when all the reactions are parallel
- In general, a structure is **geometrically unstable** if there are fewer reactive forces then equations of equilibrium.
- An unstable structure must be avoided in practice regardless of determinacy.

$r < 3n$ unstable

$r \geq 3n$ unstable if members reactions are concurrent or parallel or contains a collapsible mechanism

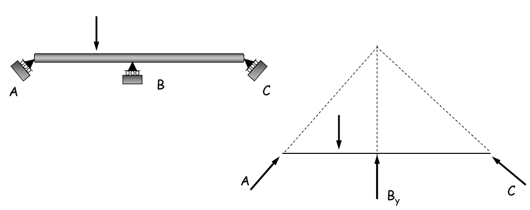
Determinacy and Stability

- Unstable - Partial Constraints

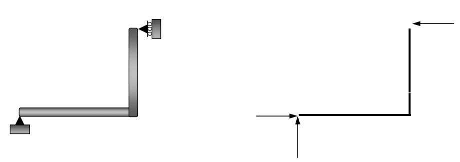


Determinacy and Stability

- Unstable - Improper Constraints

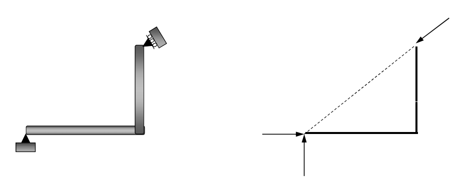


Determinacy and Stability



Stable Reactions are nonconcurrent and nonparallel

Determinacy and Stability

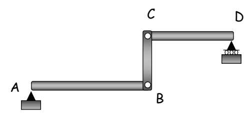


Unstable The three reactions are concurrent

Determinacy and Stability

Unstable The three reactions are parallel

Determinacy and Stability



r = 7

n = 3

r < 3n

Unstable r < 3n and member CD is free to move horizontally

Application of the Equations of Equilibrium

- **Free-Body Diagram** - disassemble the structure and draw a free-body diagram of each member.
- **Equations of Equilibrium** - The total number of unknowns should be equal to the number of equilibrium equations

Application of the Equations of Equilibrium

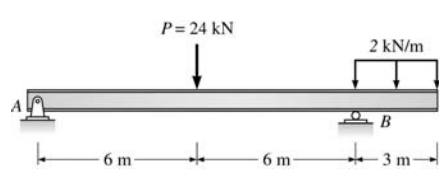
- **Free-Body Diagram**
  - Disassemble the structure and draw a free-body diagram of each member.
  - It may be necessary to supplement a member free-body diagram with a free-body diagram of the *entire structure*.
  - Remember that reactive forces common on two members act with equal magnitudes but opposite direction on their respective free bodies.
  - Identify any two-force members

Application of the Equations of Equilibrium

- **Equations of Equilibrium**
  - Check is the structure is determinate and stable
  - Attempt to apply the moment equation $\Sigma M=0$ at a point that lies at the intersection of the lines of action of as many forces as possible
  - When applying $\Sigma F_x=0$ and $\Sigma F_y=0$, orient the *x* and *y* axes along lines that will provide the simplest reduction of forces into their *x* and *y* components
  - If the solution of the equilibrium equations yields a *negative* value for an unknown, it indicates that the direction is *opposite* of that assumed

Application of the Equations of Equilibrium

- Draw the free-body diagram and determine the reactions for the following structures



Application of the Equations of Equilibrium

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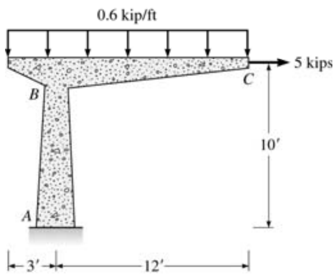

Application of the Equations of Equilibrium

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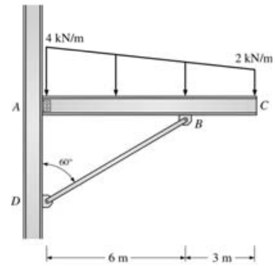

Analysis of Statically Determinate Structures

Any Questions?